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Symmetry properties of divergences of vector currents

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Abstract. We propose to investigate the degree of validity of the octet dominance formulation of the partial conservation of vector currents, recently used by several authors. To this end, possible contributions transforming like a '27' representation of the SU(3) group are assumed for the divergences of vector currents, in addition to the 'octet' part represented by scalar mesons. Their magnitude is investigated by solving the equations for the divergences of vector currents with the aid of a perturbation expansion in λ(SU(3) breaking) and e^2(SU(2) breaking). It is shown that 'octet dominance' for the SU(3) breaking of the divergences of vector currents can be safely assumed only for certain transitions, while somewhat higher '27' contributions are encountered for other matrix elements as well as in the appropriate SU(2)-breaking case. The Gell-Mann–Okubo mass formulae with electromagnetic corrections for baryons and for pseudoscalar mesons, the electromagnetic mixing angles and the Coleman–Glashow formulae are all reproduced in our solution, and in addition three new hybrid mass formulae are obtained. Finally, we discuss the domain of validity of the results emphasizing which of them turn out to hold for an arbitrary mixture of the usual electromagnetic hamiltonian and a tadpole contribution.

1. Introduction

It is well known (de Swart 1963, Clavelli 1969, Dashen 1969) that to a rather good approximation, strong and electromagnetic mass differences transform like an octet representation of SU(3). Specifically, for the hamiltonian matrix elements one has (de Swart 1963, Clavelli 1969) $\langle\alpha|H^{(27)}_{\text{strong}}|\alpha\rangle/\langle\alpha|H^{(8)}_{\text{strong}}|\alpha\rangle$ equal to approximately 14% for the $\frac{1}{2}^+$ baryon octet, 10% for the pseudoscalar meson octet (or 20% if a linear mass formula is used) and approximately 25% for the vector mesons. The appropriate figure for H_{em} is (de Swart 1963) for the baryon octet $\langle\alpha|H^{(27)}_{\text{em}}|\alpha\rangle/\langle\alpha|H^{(8)}_{\text{em}}|\alpha\rangle \simeq 20\%$. On the other hand, $H^{(8)}_{\text{em}}$ gives to lowest order $m_{\pi^+} = m_{\pi^0}$, and the observed mass difference is due (to order e^2) to the '27 term'.

During the last few years, the assumption of the 'partial conservation of vector current' (PCVC) has been used by various authors to calculate processes to which scalar particles contribute, such as K_{l3} decay (Nieh 1968, Mackey *et al* 1968, Dahmen *et al* 1968a, b, Arnowitt *et al* 1969) or η weak decay (Eliezer and Singer 1969), as well as to obtain sum rules for masses, coupling constants and second class form factors (Marshak *et al* 1966, Eliezer and Singer 1970, 1971, Schülke 1969a, b).

The PCVC hypothesis has been implemented by retaining the scalar pole contribution to appropriate dispersion relations (Nieh 1968, Mackey *et al* 1968, Dahmen *et al* 1968a, b, Schülke 1969a, b) and more recently by assuming (Arnowitt *et al* 1969, Eliezer

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and Singer 1970, 1971) that the divergences of the vector currents are proportional to an octet of interpolating scalar fields. It is therefore of obvious interest to investigate the validity of the octet dominance of the matrix elements of the divergences of the vector currents, in particular in comparison with the much discussed octet enhancement of the SU(3)-broken hamiltonian.

The divergences of the vector currents $\partial_\mu V_\mu^\alpha$ are related to the symmetry-breaking part of the hamiltonian density $H_{\mathrm{SB}}(\boldsymbol{x}, t)$ by the following relation, which holds if $H_{\mathrm{SB}}(\boldsymbol{x}, t)$ does not contain derivatives (see eg Dashen and Weinstein 1969, Renner 1969)

$$\partial_\mu V_\mu^\alpha(\boldsymbol{x}, t) = -\mathrm{i}[F^\alpha(t), H_{\mathrm{SB}}(\boldsymbol{x}, t)] \tag{1.1}$$

where $F^\alpha(t) = \int \mathrm{d}^3x V_0^\alpha(\boldsymbol{x}, t)$ is the vector charge. Thus, if an appropriate form is assumed for $H_{\mathrm{SB}}(\boldsymbol{x}, t)$, the information on its matrix elements obtained from the physical masses can be translated by the use of equation (1.1) into information on the matrix elements of $\partial_\mu V_\mu^\alpha$. The usual practice of PCVC is to assume $\partial_\mu V_\mu^\alpha \sim c_\alpha \phi^\alpha$, where ϕ^α is an octet of scalar fields and c_α are constants. It is obviously desirable to check for possible contributions from other representations and thus to verify the validity of the usual procedure, even before making the extrapolation in q^2.

In this article we consider possible additional contributions to the divergences of vector currents, having '27' transformation properties, and we estimate their magnitude relative to the octet part. To this purpose, a method is developed to obtain sum rules for masses, taking these corrections into account. In the appropriate limits our sum rules reproduce the well known mass formulae.

In order to test the possible deviation from octet dominance, we rewrite the definition of the divergences of vector currents to include '27' contributions as follows:

$$\partial_\mu V_\mu(\Delta S = 0, \Delta Q = 1) = \mathrm{i}f_{\pi_{\mathrm{N}}^+} m_{\pi_{\mathrm{N}}^+}^2 \phi_{\pi_{\mathrm{N}}^+} + R_1 \tag{1.2}$$

$$\partial_\mu V_\mu(\Delta S = 1, \Delta Q = 1) = \mathrm{i}f_{\kappa^+} m_{\kappa^+}^2 \phi_{\kappa^+} + R_2 \tag{1.3}$$

$$\partial_\mu V_\mu(\Delta S = 1, \Delta Q = 0) = \mathrm{i}f_{\kappa^0} m_{\kappa^0}^2 \phi_{\kappa^0} + R_3 \tag{1.4}$$

where $\phi_{\pi_{\mathrm{N}}}$ and ϕ_κ are the renormalized field operators that create the particles with the spin, parity and isospin quantum numbers $J^P = 0^+$, $I(\pi_{\mathrm{N}}) = 1$, $I(\kappa) = \frac{1}{2}$. f_S, where $S = \pi_{\mathrm{N}}^+, \kappa^+$ or κ^0, is defined through

$$(2\pi)^3(2q_0)^{1/2}\langle S(q)| V_\mu|0\rangle = \mathrm{i}f_S q_\mu. \tag{1.5}$$

In the limit of exact SU(2) the coupling constant $f_{\pi_{\mathrm{N}}}$ would vanish and $f_{\kappa^+} = f_{\kappa^0}$, while in the limit of exact SU(3) one has $f_{\pi_{\mathrm{N}}} = f_\kappa = 0$. We also remark that the octet assumption for the scalar mesons implies the following sum rule:

$$f_{\kappa^+} m_{\kappa^+}^2 - f_{\kappa^0} m_{\kappa^0}^2 = f_{\pi_{\mathrm{N}}^+} m_{\pi_{\mathrm{N}}^+}^2. \tag{1.6}$$

The octet part of the divergences is thus described by appropriate interpolating scalar fields. Although in the present work we do not make explicit use of the scalar meson dominance, we use the PCVC formulation of equations (1.2)–(1.4) for its convenience for possible further applications.

Our previous formulation of PCVC through dominance by scalar mesons (Eliezer and Singer 1970, 1971) is valid if the matrix elements of R are much smaller than the matrix elements of the scalar field operator terms. This possibility is investigated in this article for physical states in the vanishing momentum transfer region.

The residual operator R can be written as the sum of two terms

$$R_i = S_i + E_i \qquad (i = 1, 2, 3), \tag{1.7}$$

where S_i and E_i are induced by the SU(3) and SU(2) symmetry breaking respectively and transform as the 27 representation of SU(3).

Taking the usual octet transformation properties for the electromagnetic current, the 27 representation is the only one available for E_i to second order in the electromagnetic coupling. More general assumptions for the electromagnetic interaction allow also 10, $\overline{10}$ contributions. Nevertheless, we disregard this possibility in view of the fact that the nonappearance of 10, $\overline{10}$ is a necessary requirement for obtaining the Coleman–Glashow formula, which holds to a very good accuracy.

2. Equations for divergences of currents

In this section we establish sets of equations fulfilled by the matrix elements of the divergences of currents, for both the octet of pseudoscalar mesons and the octet of baryons (for the detailed equations see Eliezer 1971). These equations will then be solved separately by a perturbative approach in the SU(3) and SU(2) symmetry-breaking parameters, namely λ and e^2 respectively. The relationship between the solutions will take the forms of hybrid mass formulae.

The matrix elements of the vector currents V_μ^α between states of pseudoscalar mesons P_i or baryons B_i are given respectively by (we drop the octet symbol α)

$$\langle P_2(q_2)|V_\mu(x=0)|P_1(q_1)\rangle = F_1^{P_1,P_2}(q^2)\left((q_1+q_2)_\mu - \frac{q_\mu}{q^2}(m_1^2-m_2^2)\right) + F_2^{P_1,P_2}(q^2)\frac{q_\mu}{q^2} \tag{2.1}$$

$$\langle B_2(q_2)|V_\mu(x=0)|B_1(q_1)\rangle$$
$$= \bar{u}_2\left\{f_1^{B_1,B_2}(q^2)\left(\gamma_\mu - \frac{q_\mu}{q^2}(m_1-m_2)\right) + f_2^{B_1,B_2}(q^2)\frac{q_\mu}{q^2} + f_3^{B_1,B_2}(q^2)\sigma_{\mu\nu}q_\nu\right\}u_1, \tag{2.2}$$

where $q_\mu = (q_1-q_2)_\mu$. Taking the divergence of equations (2.1), (2.2) one has

$$\langle P_2(q_2)|\mathrm{i}\partial_\mu V_\mu(x=0)|P_1(q_1)\rangle = -F_2^{P_1,P_2}(q^2) \tag{2.3}$$

$$\langle B_2(q_2)|\mathrm{i}\partial_\mu V_\mu(x=0)|B_1(q_1)\rangle = -f_2^{B_1,B_2}(q^2). \tag{2.4}$$

The requirements of nonexistence of pole singularities at $q^2 = 0$ in (2.1), (2.2) relate $F_1(0)$ to $F_2(0)$ and $f_2(0)$ to $f_1(0)$ as follows:

$$F_2^{P_1,P_2}(0) = F_1^{P_1,P_2}(0)(m_1^2-m_2^2) \tag{2.5}$$

$$f_2^{B_1,B_2}(0) = f_1^{B_1,B_2}(0)(M_1-M_2) \tag{2.6}$$

and we thus finally arrive at the two sets of equations for the matrix elements of the divergences of vector currents:

$$\langle P_2(q_2)|\mathrm{i}\partial_\mu V_\mu(0)|P_1(q_1)\rangle_{q^2=0} = F_1^{P_1,P_2}(0)(m_2^2-m_1^2) \tag{2.7}$$

$$\langle B_2(q_2)|\mathrm{i}\partial_\mu V_\mu(0)|B_1(q_1)\rangle_{q^2=0} = f_1^{B_1,B_2}(0)(M_2-M_1). \tag{2.8}$$

In the next section, we solve these sets of equations to order $\mathrm{O}(\lambda)+\mathrm{O}(e^2)+\mathrm{O}(\lambda e^2)$, after expressing their left hand side with the aid of (1.2)–(1.4) and (1.7) and use of the

Wigner–Eckart theorem. In this context, we remember that the mass differences $m_2^2-m_1^2$ or M_2-M_1 are of order $\mathrm{O}(\lambda)+\mathrm{O}(\lambda^2)+\mathrm{O}(e^2)+\mathrm{O}(\lambda e^2)+\ldots$.

The consistency of the equations (2.7), (2.8) to order e^2 requires that we take into account π^0–η and Σ^0–Λ mixing (Dalitz and Von Hippel 1964, Matsuda *et al* 1969). We thus define two mixing angles, ϕ and θ respectively, by the relations

$$\begin{pmatrix}\pi^0\\ \eta^0\end{pmatrix}=\begin{pmatrix}\cos\phi & \sin\phi\\ -\sin\phi & \cos\phi\end{pmatrix}\begin{pmatrix}\pi'\\ \eta'\end{pmatrix} \tag{2.9}$$

$$\begin{pmatrix}\Sigma^0\\ \Lambda^0\end{pmatrix}=\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\Sigma'\\ \Lambda'\end{pmatrix} \tag{2.10}$$

π', η', Σ', Λ' being the SU(2) eigenstates. Thus, whenever π^0, η^0, Σ^0 and Λ^0 appear in the matrix elements of equations (2.7) and (2.8) the expressions used are

$$\left\langle\begin{pmatrix}\cos\phi & \sin\phi\\ -\sin\phi & \cos\phi\end{pmatrix}\begin{pmatrix}\pi'\\ \eta'\end{pmatrix}\middle| \mathrm{i}\partial_\mu V_\mu \middle| P\right\rangle$$
$$=\begin{pmatrix}(m_{\pi^0}^2-m_P^2)\cos\phi & (m_{\pi^0}^2-m_P^2)\sin\phi\\ -(m_\eta^2-m_P^2)\sin\phi & (m_\eta^2-m_P^2)\cos\phi\end{pmatrix}\begin{pmatrix}F_1^{P,\pi'}\\ F_1^{P,\eta'}\end{pmatrix} \tag{2.11}$$

and

$$\left\langle\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\Sigma'\\ \Lambda'\end{pmatrix}\middle| \mathrm{i}\partial_\mu V_\mu \middle| B\right\rangle$$
$$=\begin{pmatrix}(m_{\Sigma^0}-m_B)\cos\theta & (m_{\Sigma^0}-m_B)\sin\theta\\ -(m_\Lambda-m_B)\sin\theta & (m_\Lambda-m_B)\cos\theta\end{pmatrix}\begin{pmatrix}f_1^{B,\Sigma'}\\ f_1^{B,\Lambda'}\end{pmatrix}. \tag{2.12}$$

3. Solution of the equations for divergences of currents

3.1. General

We find it convenient at this point to formulate our problem by considering the symmetry-breaking hamiltonian H_{SB}, which, under the conditions mentioned in the introduction, is related to $\partial_\mu V_\mu$ by equation (1.1). The H_{SB} embodying the assumptions outlined in the introduction is

$$H_{\mathrm{SB}} = g_8 H^{(8)}_{(0,0,0)}+g_{27}H^{(27)}_{(0,0,0)}+\alpha_8 H^{(8)}_{(0,0,0)}+\alpha_{27}H^{(27)}_{(0,0,0)}$$
$$+\beta_8 H^{(8)}_{(0,1,0)}+\beta_{27}H^{(27)}_{(0,1,0)}+\gamma_{27}H^{(27)}_{(0,2,0)}. \tag{3.1}$$

g_8, g_{27} are assumed to be of order λ, the SU(3) symmetry-breaking parameter, while α_i, β_i, γ_i are of the order of the SU(2) symmetry-breaking parameter. The notation is $H^{(N)}_{(Y,I,I_3)}$ where N refers to the irreducible representation to which the operator belongs.

Using (1.1) and (3.1) one can re-express the right hand side of equations (1.2)–(1.4) as follows:

$$f_{\pi_N^+}m_{\pi_N^+}^2\phi_{\pi_N^+} = \beta_8 H^{(8)}_{(0,1,-1)} \tag{3.2}$$

$$-f_{K^+}m_{K^+}^2\phi_{K^+} = (\sqrt{\tfrac{3}{4}}g_8+\sqrt{\tfrac{3}{4}}\alpha_8+\tfrac{1}{2}\beta_8)H^{(8)}_{(-1,1/2,-1/2)} \tag{3.3}$$

$$-f_{K^0}m_{K^0}^2\phi_{K^0} = (\sqrt{\tfrac{3}{4}}g_8+\sqrt{\tfrac{3}{4}}\alpha_8-\tfrac{1}{2}\beta_8)H^{(8)}_{(-1,1/2,1/2)} \tag{3.4}$$

$$R_1 = \beta_{27}H^{(27)}_{(0,1,-1)}+\sqrt{3}\gamma_{27}H^{(27)}_{(0,2,-1)} \tag{3.5}$$

$$R_2 = (\sqrt{2}g_{27}+\sqrt{2}\alpha_{27}+\sqrt{\tfrac{2}{3}}\beta_{27})H^{(27)}_{(-1,1/2,-1/2)}+(\sqrt{\tfrac{5}{6}}\beta_{27}+\sqrt{\tfrac{1}{2}}\gamma_{27})H^{(27)}_{(-1,3/2,-1/2)} \tag{3.6}$$

$$R_3 = (\sqrt{2}g_{27}+\sqrt{2}\alpha_{27}-\sqrt{\tfrac{2}{3}}\beta_{27})H^{(27)}_{(-1,1/2,1/2)}+(\sqrt{\tfrac{5}{6}}\beta_{27}-\sqrt{\tfrac{1}{2}}\gamma_{27})H^{(27)}_{(-1,3/2,1/2)}. \tag{3.7}$$

For further reference, we also define the reduced matrix element for the meson and baryon systems

$$d^N_{(s)} \equiv \xi_2\sqrt{\frac{N}{8}}\langle P(8)||N||P(8)\rangle_{(s)} \tag{3.8}$$

$$D^N_{s,A} \equiv \xi_2\sqrt{\frac{N}{8}}\langle B(8)||N||B(8)\rangle_{s,A} \tag{3.9}$$

where $\xi_2 = \pm 1$ as defined in de Swart (1963) and N is the dimension of the irreducible representation to which the operator $H^N_{(Y,I,I_3)}$ belongs.

3.2. *The pseudoscalar mesons system*

We proceed now to solve the set of equations (2.7). As the equations obtained are extensive and their explicit form is not needed in order to follow the argument, we shall only describe here our method of solution.

The left hand side of (2.7) is expressed using (1.2)–(1.4) and (3.2)–(3.7) as well as the Wigner–Eckart theorem. For the right hand side we need to use to the order we solve, only the SU(3) values of the F_1 and f_1. The equations are expressed to order $O(\lambda)+O(e^2)+O(\lambda e^2)$, after taking into account the electromagnetic mixing (2.11) and (2.12). We thus obtain a set of nine equations for the three transitions considered in equations (1.2)–(1.4), three for each. For easier orientation, we record here as an example the three equations obtained for $\partial_\mu V_\mu(\Delta S = 0, \Delta Q = 1)$:

$$\sqrt{\tfrac{3}{10}}\beta_8 d^8_s-\sqrt{\tfrac{1}{5}}\beta_{27}d^{27} = (m^2_{K^0}-m^2_{K^+})$$
$$\sqrt{\tfrac{3}{2}}\gamma_{27}d^{27} = \sqrt{2}(m^2_{\pi^+}-m^2_{\pi^0})$$
$$\sqrt{\tfrac{1}{5}}\beta_8 d^8_s+\sqrt{\tfrac{3}{10}}\beta_{27}d^{27} = \sqrt{2}\tan\phi(m^2_\eta-m^2_\pi).$$

In the last equation above, there is no need to identify the pion charge as ϕ is of order $O(e^2)$ and the explicit pion charge would appear in this equation only if keeping terms to second order in $(SU_2)_I$-symmetry breaking, that is $O(e^4)$. There is likewise no need to distinguish between m_{η^0} and m_η—the mass before the electromagnetic mass shift. We also point out that the second equation gives directly the magnitude of the $\Delta I = 2$ contribution to $\partial_\mu V_\mu(\Delta S = 0, \Delta Q = 1)$ induced by $H^{(27)}$, namely

$$\langle\pi^0|\partial_\mu V_\mu(\Delta S = 0, \Delta Q = 1)|\pi^+\rangle = \sqrt{\tfrac{3}{2}}\gamma_{27}d^{27} = \sqrt{2}(m^2_{\pi^+}-m^2_{\pi^0}) = 18{\cdot}4\times 10^{-4}\,(\text{GeV})^2. \tag{3.10}$$

Neglecting the SU(2) breaking, the set of nine equations reduces to a set of two equations for the two unknowns $g_8 d^8_s$ and $g_{27}d^{27}$, whose solution is†

$$g_8 d^8_s = \sqrt{\tfrac{2}{5}}(2m^2_K+m^2_\eta-3m^2_\pi) \simeq 0{\cdot}462\,(\text{GeV})^2 \tag{3.11}$$

$$g_{27}d^{27} = \sqrt{\tfrac{3}{20}}(4m^2_K-3m^2_\eta-m^2_\pi) \simeq 0{\cdot}024\,(\text{GeV})^2. \tag{3.12}$$

† The sign $\simeq$ is used in the formulae in which the average mass of the SU(2) multiplet is employed for the numerical calculation, that is before solving the equations with SU(2) breaking included.

In the limit of vanishing '27' contribution, the Gell-Mann–Okubo mass formula results from (3.12).

We are now in the position to express the relative contributions of '8' and '27' to $\partial_\mu V_\mu^{\Delta s=1}$, which turns out to be

$$\frac{\langle \partial V\rangle_{27}^{\Delta s=1,\pi K}}{\langle \partial V\rangle_{8}^{\Delta s=1,\pi K}} = \frac{(\frac{1}{30})^{1/2} g_{27} d^{27}}{(\frac{9}{80})^{1/2} g_8 d_s^8} \simeq 2\cdot 8\,\% \tag{3.13}$$

where $\langle \partial V\rangle_i^{\Delta s=1,\pi K} \equiv \langle \pi|\partial_\mu V_\mu|K\rangle_i^{\Delta s=1}$. On the other side, for the η–K transition $\langle \partial V\rangle_i^{\Delta s=1,\eta K} \equiv \langle \eta|\partial_\mu V_\mu|K\rangle_i^{\Delta s=1}$ one has

$$\frac{\langle \partial V\rangle_{27}^{\Delta s=1,\eta K}}{\langle \partial V\rangle_{8}^{\Delta s=1,\eta K}} = \frac{(\frac{9}{10})^{1/2} g_{27} d^{27}}{(\frac{3}{80})^{1/2} g_8 d_s^8} \simeq 25\cdot 5\,\%. \tag{3.14}$$

We can now use the solution (3.11), (3.12) to rewrite our original set of equations as an homogeneous set to order $O(e^2)$ and $O(\lambda e^2)$ for the general case of both SU(3) and SU(2) breaking. In addition to the separate equation (3.10) which gives us directly $\gamma_{27} d^{27}$ we find that by rearrangement we obtain only four more independent equations for the eight unknowns $\alpha_8 d_s^8$, $\alpha_{27} d^{27}$, m_π^2, m_K^2, m_η^2, ϕ, $\beta_8 d_s^8$ and $\beta_{27} d^{27}$. The fact that there are finally only five independent equations is related to the occurrence of only five independent mass differences for the set of equations (2.7). The five independent equations are (3.10) and the following four equations:

$$m_{K^0}^2 - m_{K^+}^2 = 2\sqrt{3}\tan\phi(m_\eta^2 - m_\pi^2) - \sqrt{\tfrac{3}{10}}\beta_8 d_s^8 - 4\sqrt{\tfrac{1}{5}}\beta_{27} d^{27} \tag{3.15}$$

$$m_{K^0}^2 - m_{K^+}^2 = \sqrt{\tfrac{3}{10}}\beta_8 d_s^8 - \sqrt{\tfrac{1}{5}}\beta_{27} d^{27} \tag{3.16}$$

$$m_{K^0}^2 + m_{K^+}^2 - 2m_K^2 - 2(m_{\eta^0}^2 - m_\eta^2) = -\sqrt{\tfrac{1}{10}}\alpha_8 d_s^8 + 6\sqrt{\tfrac{1}{15}}\alpha_{27} d^{27} \tag{3.17}$$

$$m_{K^0}^2 + m_{K^+}^2 - 2m_K^2 + 2\{m_\pi^2 - (\tfrac{1}{3}m_{\pi^0}^2 + \tfrac{2}{3}m_{\pi^+}^2)\} = 3\sqrt{\tfrac{1}{10}}\alpha_8 d_s^8 + 2\sqrt{\tfrac{1}{15}}\alpha_{27} d^{27}. \tag{3.18}$$

In order to obtain a solution, we must resort to some further relations between the various α, β and γ. A natural assumption is to use U spin invariance of the SU(2) breaking interaction. In doing so, for both the '8' and '27' contributions one obtains

$$\beta_8 = \sqrt{3}\alpha_8, \qquad \beta_{27} = \sqrt{3}\alpha_{27}, \qquad \gamma_{27} = \sqrt{5}\alpha_{27}. \tag{3.19}$$

In the last section, we shall discuss some other possibilities and their solutions.

The three conditions (3.19) are not yet sufficient for obtaining a complete solution for the eight unknowns in equations (3.15)–(3.18). We choose therefore to treat $m_{\eta^0}^2 - m_\eta^2$ as a known quantity in solving the equations and as it will be immediately apparent, its knowledge is in fact not needed in order to obtain the relations of interest here (Brown *et al* 1969)†. Using (3.19) with (3.15)–(3.18) we get the solution

$$\tan\phi = \frac{(m_{K^0}^2 - m_{K^+}^2) + (m_{\pi^+}^2 - m_{\pi^0}^2)}{\sqrt{3}(m_\eta^2 - m_\pi^2)} = 0\cdot 011 \pm 0\cdot 001 \tag{3.20}$$

$$\alpha_8 d_s^8 = \sqrt{\tfrac{10}{9}}\{(m_{K^0}^2 - m_{K^+}^2) + \tfrac{2}{3}(m_{\pi^+}^2 - m_{\pi^0}^2)\} = (46\cdot 2 \pm 1\cdot 5)\times 10^{-4}\,(\mathrm{GeV})^2 \tag{3.21}$$

$$\alpha_{27} d^{27} = 2\sqrt{\tfrac{1}{15}}(m_{\pi^+}^2 - m_{\pi^0}^2) = (6\cdot 7 \pm 0\cdot 0)\times 10^{-4}\,(\mathrm{GeV})^2 \tag{3.22}$$

† These authors estimate the electromagnetic η mass shift using vector meson dominance of the hadronic electromagnetic current which gives $m_{\eta^0} - m_\eta \simeq -1$ MeV.

$$m_{\mathrm{K}}^2 = \tfrac{1}{3}(2m_{\mathrm{K}^0}^2+m_{\mathrm{K}^+}^2)-\tfrac{1}{3}(m_{\pi^+}^2-m_{\pi^0}^2)-(m_{\eta^0}^2-m_{\eta}^2) \tag{3.23}$$

$$m_{\pi}^2 = \tfrac{1}{3}(2m_{\pi^+}^2+m_{\pi^0}^2)+\tfrac{2}{3}(m_{\mathrm{K}^0}^2-m_{\mathrm{K}^+}^2)-(m_{\eta^0}^2-m_{\eta}^2). \tag{3.24}$$

As it is obvious from (3.20)–(3.24), the value of m_η does not appear in the quantities of interest (3.20)–(3.22). Furthermore, from (3.23) and (3.24) and the Okubo–Gell-Mann mass formula which obtains in (3.12) when $g_{27} = 0$ we deduce the mass formula with electromagnetic corrections included†:

$$2(m_{\mathrm{K}^+}^2+m_{\mathrm{K}^0}^2)-(2m_{\pi^+}^2-m_{\pi^0}^2)-3m_{\eta^0}^2 = 0. \tag{3.25}$$

Our formula agrees with the derivation of Dalitz and Sutherland (1965a, b) but not with Okubo's suggestion (Okubo 1964) of using neutral masses only. For the mixing η–π^0 angle we get the same result as other authors, who obtain it (Okubo and Sakita 1963) by diagonalizing the η–π mass matrix.

The relative contributions of '8' and '27' to $\partial_\mu V_\mu(\Delta S = 0, \Delta Q = 1)$ transitions are now obtainable

$$\frac{\langle \partial V\rangle_{27}^{\Delta s=0,\mathrm{K}}}{\langle \partial V\rangle_{8}^{\Delta s=0,\mathrm{K}}} = \frac{-(\frac{1}{5})^{1/2}\beta_{27}d^{27}}{(\frac{3}{10})^{1/2}\beta_8 d_s^8} = -12\% \tag{3.26}$$

where

$$\langle \partial V\rangle_{i}^{\Delta s=0,\mathrm{K}} = \langle \mathrm{K}^+|\partial_\mu V_\mu|\mathrm{K}^0\rangle_i, \qquad i = 8, 27.$$

3.3. *The baryon system*

In this case we have to solve the set of equations (2.8). The same procedure as described in § 3.2 is being used, therefore we omit repeating the description of the various steps. Writing the equations for the transitions (1.2)–(1.4) we obtain a set of seventeen equations. Our first step is to neglect SU(2) breaking and then the set of equations reduces to three independent equations for $g_8D_A^8$, $g_8D_S^8$ and $g_{27}D^{27}$, whose solution gives

$$g_8D_A^8 = \sqrt{2}(m_{\mathrm{N}}-m_{\Xi}) \simeq -530{\cdot}6\ \mathrm{MeV} \tag{3.27}$$

$$g_8D_s^8 = \sqrt{\tfrac{2}{5}}(m_{\Xi}+m_{\mathrm{N}}+m_{\Lambda}-3m_{\Sigma}) \simeq -130{\cdot}4\ \mathrm{MeV} \tag{3.28}$$

$$g_{27}D^{27} = \sqrt{\tfrac{3}{20}}(2m_{\mathrm{N}}+2m_{\Xi}-3m_{\Lambda}-m_{\Sigma}) \simeq -9{\cdot}6\ \mathrm{MeV}. \tag{3.29}$$

From these equations we obtain the contributions of '27' and '8' to the matrix elements of $\partial_\mu V_\mu(\Delta S = 1)$ as follows:

$$\frac{\langle \partial V\rangle_{27}^{\Delta s=1,\mathrm{N}\Sigma}}{\langle \partial V\rangle_{8}^{\Delta s=1,\mathrm{N}\Sigma}} = \frac{(\frac{1}{15})^{1/2}g_{27}D^{27}}{(\frac{9}{40})^{1/2}g_8D_s^8+(\frac{1}{8})^{1/2}g_8D_A^8} \simeq 1{\cdot}0\% \tag{3.30}$$

where

$$\langle \partial V\rangle_{i}^{\Delta s=1,\mathrm{N}\Sigma} \equiv \langle \mathrm{N}|\partial_\mu V_\mu|\Sigma\rangle_i^{\Delta s=1}, \qquad i = 8, 27,$$

and

$$\frac{\langle \partial V\rangle_{27}^{\Delta s=1,\mathrm{N}\Lambda}}{\langle \partial V\rangle_{8}^{\Delta s=1,\mathrm{N}\Lambda}} = \frac{(\frac{9}{10})^{1/2}g_{27}D^{27}}{-(\frac{3}{80})^{1/2}g_8D_s^8+(\frac{3}{16})^{1/2}g_8D_A^8} \simeq 4{\cdot}5\% \tag{3.31}$$

† It should be clear η is taken here as the member of the octet as we do not specifically treat the singlet–octet mixing. Inclusion of η–x^0 mixing, in addition to the '27' breaking, would result in too large a number of unknowns and our equations would then be undetermined. In fact, to the best of our knowledge, there is no satisfactory way of obtaining the separate effects of these two kinds of breaking, if both are allowed to be present.

where $\langle \partial V \rangle_i^{\Delta s=1,\mathrm{NA}} \equiv \langle \mathrm{N}|\partial_\mu V_\mu|\Lambda\rangle_i^{\Delta s=1}$. The magnitude of all other possible $\Delta S = 1$ transitions lies between the above two extremum values.

We return now to the original set of seventeen equations after inserting the solutions (3.27)–(3.29), and we are dealing now with a homogeneous set of equations to order e^2 and λe^2. The five equations related to the $\Delta S = 0$, $\Delta Q = 1$ transitions allow us to solve directly for

$$\beta_8 D_A^8 = \sqrt{\tfrac{3}{2}}(m_{\Sigma^+} - m_{\Sigma^-}) = (-9{\cdot}8 \pm 0{\cdot}2)\ \mathrm{MeV} \tag{3.32}$$

$$\gamma_{27} D^{27} = \sqrt{\tfrac{1}{3}}(m_{\Sigma^-} + m_{\Sigma^-} - 2m_{\Sigma^0}) = (1{\cdot}1 \pm 0{\cdot}1)\ \mathrm{MeV} \tag{3.33}$$

as well as to obtain one sum rule

$$m_{\Sigma^-} - m_{\Sigma^+} = m_{\mathrm{n}} - m_{\mathrm{p}} + m_{\Xi^-} - m_{\Xi^0}, \tag{3.34}$$

which is the well known Coleman–Glashow relation (Coleman and Glashow 1961). We then find that the rest of the equations can be reduced to five independent equations for the remaining eight unknowns m_{N}, m_Σ, m_Ξ, m_Λ, $\alpha_{27}D^{27}$, $\alpha_8 D_s^8$, $\beta_{27}D^{27}$, θ as follows

$$\sqrt{\tfrac{1}{5}}\beta_8 D_s^8 + \sqrt{\tfrac{3}{10}}\beta_{27} D^{27} = -\sqrt{2}\tan\theta(m_\Sigma - m_\Lambda) \tag{3.35}$$

$$-\sqrt{\tfrac{3}{10}}\beta_8 D_s^8 + \sqrt{\tfrac{1}{5}}\beta_{27} D^{27} = m_{\mathrm{p}} - m_{\mathrm{n}} + \tfrac{1}{2}(m_{\Sigma^-} - m_{\Sigma^+}) \tag{3.36}$$

$$\sqrt{\tfrac{9}{10}}\alpha_8 D_s^8 + \sqrt{\tfrac{1}{2}}\alpha_8 D_A^8 + 2\sqrt{\tfrac{1}{15}}\alpha_{27} D^{27} = \{2m_\Sigma - \tfrac{2}{3}(m_{\Sigma^+} + m_{\Sigma^-} + m_{\Sigma^0})\} + (m_{\mathrm{p}} + m_{\mathrm{n}} - 2m_{\mathrm{N}}) \tag{3.37}$$

$$\sqrt{\tfrac{1}{10}}\alpha_8 D_s^8 - \sqrt{\tfrac{1}{2}}\alpha_8 D_A^8 - 2\sqrt{\tfrac{3}{5}}\alpha_{27} D^{27} = 2(m_{\Lambda^0} - m_\Lambda) + \{2m_{\mathrm{N}} - (m_{\mathrm{p}} + m_{\mathrm{n}})\} \tag{3.38}$$

$$\sqrt{\tfrac{1}{10}}\alpha_8 D_s^8 + \sqrt{\tfrac{1}{2}}\alpha_8 D_A^8 - 2\sqrt{\tfrac{3}{5}}\alpha_{27} D^{27} = 2(m_{\Lambda^0} - m_\Lambda) + \{2m_\Xi - (m_{\Xi^-} + m_{\Xi^0})\}. \tag{3.39}$$

In order to solve these equations we make the assumption of $(\mathrm{SU}(2))_{\mathrm{U}}$ invariance implying relations (3.19). Although the structure of our equations does not allow us to calculate m_Λ, its value is not needed to obtain the relations of interest. The solution is:

$$\tan\theta = \frac{(m_{\Sigma^0} - m_{\Sigma^-}) - (m_{\mathrm{n}} - m_{\mathrm{p}})}{\sqrt{3}(m_\Sigma - m_\Lambda)} = 0{\cdot}019 \pm 0{\cdot}001 \tag{3.40}$$

$$\alpha_8 D_s^8 = \sqrt{\tfrac{10}{9}}\{(m_{\mathrm{n}} - m_{\mathrm{p}}) + \tfrac{1}{10}(7m_{\Sigma^+} - 3m_{\Sigma^-} - 4m_{\Sigma^0})\} = (-2{\cdot}42 \pm 0{\cdot}13)\ \mathrm{MeV} \tag{3.41}$$

$$m_{\mathrm{N}} = \tfrac{1}{2}(m_{\mathrm{n}} + m_{\mathrm{p}}) + \tfrac{1}{6}(m_{\Sigma^-} + 2m_{\Sigma^0} - 3m_{\Sigma^+} + m_{\Xi^0} - m_{\Xi^-}) - (m_{\Lambda^0} - m_\Lambda) \tag{3.42}$$

$$m_\Xi = \tfrac{1}{2}(m_{\Xi^-} + m_{\Xi^0}) + \tfrac{1}{6}(m_{\Sigma^+} + 2m_{\Sigma^0} - 3m_{\Sigma^-} + m_{\mathrm{n}} - m_{\mathrm{p}}) - (m_{\Lambda^0} - m_\Lambda) \tag{3.43}$$

$$m_\Sigma = \tfrac{1}{3}(m_{\Sigma^+} + m_{\Sigma^0} + m_{\Sigma^-}) + \tfrac{1}{3}(m_{\mathrm{n}} - m_{\mathrm{p}} + m_{\Xi^0} - m_{\Xi^-}) - (m_{\Lambda^0} - m_\Lambda) \tag{3.44}$$

$$\frac{3(m_{\Sigma^-} - m_{\Sigma^+})}{m_\Xi - m_{\mathrm{N}}} = \frac{10(m_{\mathrm{n}} - m_{\mathrm{p}}) + (7m_{\Sigma^+} - 3m_{\Sigma^-} - 4m_{\Sigma^0})}{m_\Xi + m_{\mathrm{N}} + m_\Lambda - 3m_\Sigma}. \tag{3.45}$$

Our solution for $\tan\theta$ agrees with that of Dalitz and von Hippel (1964). Combining equations (3.42)–(3.44) by using the Gell-Mann–Okubo mass formula which obtains in (3.29) if $g_{27}D^{27} = 0$, we obtain the mass formula for the baryon octet with electromagnetic corrections included, namely

$$(m_{\mathrm{n}} + m_{\mathrm{p}}) + (m_{\Xi^-} + m_{\Xi^0}) - (m_{\Sigma^+} + m_{\Sigma^-} - m_{\Sigma^0}) - 3m_{\Lambda^0} = 0. \tag{3.46}$$

This expression agrees with that previously derived by Nauenberg (1964). Concerning now the contribution of '27' to the $\Delta S = 0$, $\Delta I = 1$ transitions, we have

$$\frac{\langle \partial V \rangle_{27}^{\Delta s=0,\mathrm{N}}}{\langle \partial V \rangle_{8}^{\Delta s=0,\mathrm{N}}} = \frac{\sqrt{\frac{1}{5}}\beta_{27}D^{27}}{-\sqrt{\frac{3}{10}}\beta_8 D_s^8 + \sqrt{\frac{1}{6}}\beta_8 D_A^8} = -11{\cdot}6\% \tag{3.47}$$

where $\langle \partial V \rangle_i^{\Delta s=0,\mathrm{N}} \equiv \langle \mathrm{p}|\partial_\mu V_\mu|\mathrm{n}\rangle_i$. The similar ratio for transition between Ξ^- and Ξ^0 states is 8%. Other transitions, involving also $\Delta I = 2$ are calculable but of less interest concerning the PCVC formulation. In addition we derive a new sum rule given in equation (3.45), where using the experimental masses one has $(6{\cdot}6 \pm 0{\cdot}3) \times 10^{-2}$ for the left hand side and $(12{\cdot}0 \pm 0{\cdot}6) \times 10^{-2}$ for the right hand side.

4. Summary and discussion

In this work, we have assumed that the contributions to the divergences of vector currents come from an octet of scalar mesons as well as from additional possible '27' contributions, our aim being to check the validity of the usual PCVC assumption. The equations obtained for the matrix elements of the divergences of current between single-particle states at zero momentum transfer ((2.7) and (2.8)) were solved in a perturbation approach to order λ and e^2. As contributions from higher order terms were neglected, our conclusions on the relative magnitudes of '8' and '27' contributions to symmetry breaking hold only if this is a valid framework. We summarize now the results obtained in the previous sections.

(i) The relative contributions of '27' and '8' are given in equations (3.13), (3.14), (3.26) for the meson system and in equations (3.30). (3.31), (3.47) for the baryon system. It appears that octet dominance is a fair approximation for both SU(3) and SU(2) breaking in both systems, although sometimes the '27' contributions are quite significant, and this already at $q^2 = 0$.

(ii) Our solution gives at the same time the values of the mixing angles for η–π^0 and Σ^0–Λ (equations (3.20) and (3.40) respectively), in agreement with previous derivations.

(iii) Assuming negligible SU(3) breaking '27' contribution, insertion of our solutions into the Gell-Mann–Okubo mass formulae thus obtained (equations (3.12), (3.29)) leads to Gell-Mann–Okubo mass formulae with electromagnetic corrections included, namely equations (3.25) and (3.46). These equations are thus shown to hold in the presence of '8' and '27' breaking for the electromagnetic interaction. It should, however, be remembered that we have used U spin invariance of the SU(2) breaking interaction in our derivation.

(iv) Two additional relations are obtained within the framework of our solution. The Coleman and Glashow (1961) relation, equation (3.34), which is derived in the presence of '27' breaking for both the SU(3) and SU(2) interactions, and the relation (Coleman and Glashow 1961)

$$m_{\Sigma^0} = \tfrac{1}{2}(m_{\Sigma^+} + m_{\Sigma^-}) \tag{4.1}$$

which obtains (equation (3.33)) if the '27' contribution to the SU(2)-breaking interaction vanishes and independently of the presence of similar contributions in the SU(3)-breaking interaction.

The use of the same general symmetry-breaking hamiltonian (3.1) for both the meson and baryon systems allows us to derive also mass formulae of the type named 'hybrid' by Coleman and Glashow (1964). Equating appropriately the ratios of $\alpha_8 D_A^8/g_8 D_A^8$, $\alpha_8 D_s^8/g_8 D_s^8$, $\alpha_{27} D^{27}/g_{27} D^{27}$ which we obtained for the baryon system to those of $\alpha_8 d_s^8/g_8 d_s^8$, $\alpha_{27} d^{27}/g_{27} d^{27}$ of the meson system we obtain three mass formulae

$$\frac{1}{3}\left(\frac{5(m_{K^0}^2-m_{K^+}^2)+2(m_{\pi^+}^2-m_{\pi^0}^2)}{2m_K^2+m_\eta^2-3m_\pi^2}\right)=\frac{1}{2}\left(\frac{m_{\Sigma^-}-m_{\Sigma^+}}{m_\Xi-m_N}\right) \tag{4.2}$$

$$\frac{5(m_{K^0}^2-m_{K^+}^2)+2(m_{\pi^+}^2-m_{\pi^0}^2)}{2m_K^2+m_\eta^2-3m_\pi^2}=\frac{5(m_n-m_p)+\frac{1}{2}(7m_{\Sigma^+}-3m_{\Sigma^-}-4m_{\Sigma^0})}{m_\Xi+m_N+m_\Lambda-3m_\Sigma} \tag{4.3}$$

$$\frac{2(m_{\pi^+}^2-m_{\pi^0}^2)}{4m_K^2-3m_\eta^2-m_\pi^2}=\frac{m_{\Sigma^-}+m_{\Sigma^+}-2m_{\Sigma^0}}{2m_N+2m_\Xi-3m_\Lambda-m_\Sigma}. \tag{4.4}$$

The comparison of equations (4.2) and (4.3) produces our previous sum rule given in equation (3.45). Using the experimental masses, we find (4.2) to be *exact* within the experimental error. For (4.3) one has $(3\cdot3\pm0\cdot1)\times10^{-2}$ for the left hand side and $(6\cdot0\pm0\cdot3)\times10^{-2}$ for the right hand side. These are very remarkable results. As it was already stressed by Coleman and Glashow (1964) who previously derived hybrid formulae of somewhat less impressive validity, *a priori* these formulae might fail by order of magnitude. The formulae of Coleman and Glashow differ from ours as theirs were derived within the more limited framework of the tadpole model.

In our third hybrid formula (4.4) one has for the left hand side $(4\cdot2\pm0\cdot0)\times10^{-2}$ and for the right hand side $(-4\cdot35\pm0\cdot14)\times10^{-2}$. The disagreement here is not surprising and is most likely due to our neglect of η–X mixing. In our solution, the denominator of the left hand side of (4.4) is attributed wholly to '27' breaking (see equation (3.12)), while inclusion of η–X mixing can easily change the sign of the '27' contribution.

It should be stressed that from the conclusions summarized above, only those under (i) refer directly to the PCVC assumption. The results (ii), (iii) and (4.1) are of more general validity and were obtained using the symmetry properties listed and the perturbation formulation. The results (4.2) and (4.3), although a direct result of PCVC, would also follow in any model which requires the use of the same hamiltonian (3.1) for both the meson and baryon system. The latter requirement leads then also to (4.4).

Finally, we should like to discuss to what extent some of our results are related to the set of relations (3.19).

Firstly, the Coleman–Glashow tadpole model results (Coleman and Glashow 1964) are reproduced by assuming $\alpha_8=\alpha_{27}=\beta_{27}=\gamma_{27}=0$, $\beta_8\neq0$. It is possible to obtain Gell-Mann–Okubo type formulae with electromagnetic corrections also in this case, however, due to the lack of the '27' contribution they read

$$2(m_{K^+}^2+m_{K^0}^2)-m_\pi^2-3m_\eta^2=0$$

and

$$(m_n+m_p)+(m_{\Xi^-}-m_{\Xi^0})-m_{\Sigma^0}-3m_\Lambda=0.$$

Secondly, if we make the assumption (3.19) *except* the relation $\beta_8=\sqrt{3}\alpha_8$, that is, we leave α_8, β_8 independent, we can still obtain the mass formulae (3.25), (3.34) and (3.46). This shows these results to be valid for an arbitrary mixture of the 'usual' electromagnetic hamiltonian and tadpole contributions. Likewise, the results for the angles θ, ϕ still hold for such mixture. Furthermore the essential conclusions on the magnitudes of $\langle\partial V\rangle$ are unchanged if one assumes $\alpha_8=0$, that is, tadpole + '27' contributions only.

In concluding, we emphasize that our method enables one to study the effects of SU(3) and SU(2) breaking within a unified formalism and is transparent enough to allow one to investigate easily on the domain of validity of the various formulae obtained.

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